AUSD Board of Education Mathematics Workshop

AUSD | Presented by Phil Gonsalves | Sept. 18, 2012

1

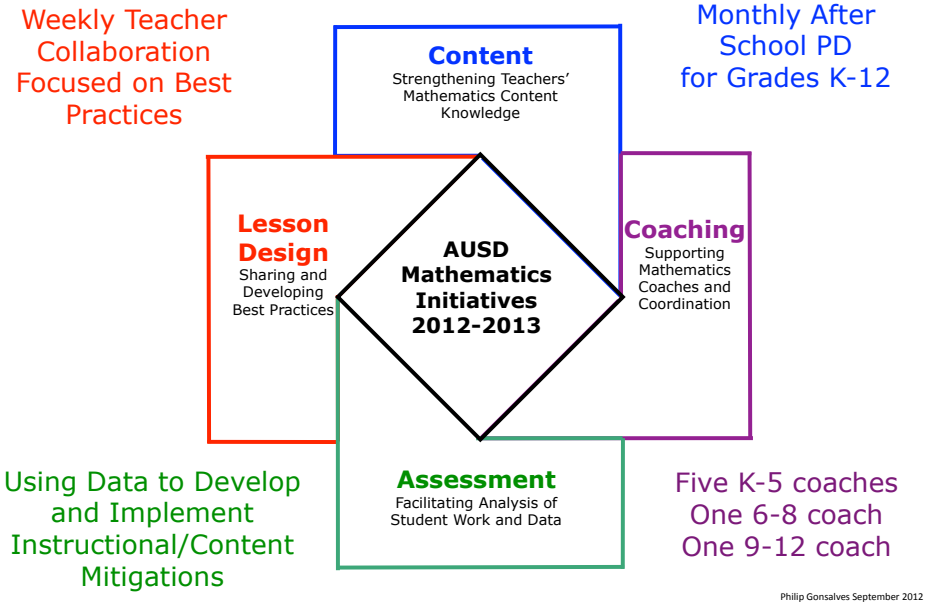
A Few Notes:

Last year SIMI received the CSBA Golden Bell Award.

SIMI has been selected for inclusion in the Magna Best Practices Database, one of only 81 "highly-scored programs selected for this special recognition from the 300 nominations" received this past year.

2

AUSD Mathematics Action



Updates on AUSD Mathematics Initiatives

- Summer and school year Professional Development
- Mathematics coaching support for all AUSD teachers
- Mathematics Achievement Academies (MAA) ongoing and could include incoming Grade 6 and Grade 7 students in summer 2013
- Common Core (CC) implementation support including hyperlinked pacing guides and benchmark assessments correlated to both 1997 CA standards and CC standards

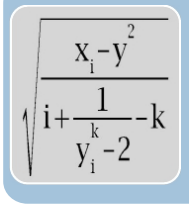
New Focus for California Common Core State Standards in Mathematics

- Aim for clarity and specificity
  - Focus on key topics at each grade level
  - Coherent progressions across grade levels
- Balance of concepts and skills
  - CCSS require both conceptual understanding and procedural skills
- Mathematical Practices
  - Foster reasoning and sense making in mathematics
- College and Career Readiness incorporated
- Internationally benchmarked

Design & Organization of Mathematics CCSS

Standards for Mathematical Practice

- **Carry across all grade levels**
- **Describe mathematical habits of mind that should be taught explicitly to all students**



Standards for Mathematical Content

- **K-8 standards are presented by grade level**
- **Organized into domains that progress over several grades**
- **Grade introductions give 2-4 focal points at each grade level**
- **High school standards are presented by conceptual theme**

Standards for Mathematical Practice

- **Make sense of problems and persevere in solving them**
- **Reason abstractly and quantitatively**
- **Construct viable arguments and critique the reasoning of others**
- **Model with mathematics**
- **Use appropriate tools strategically**
- **Attend to precision**
- **Look for and make use of structure**
- **Look for and express regularity in repeated reasoning**

Standards for Mathematical Practice

21st Century Skills

- Communication
- Collaboration
- Creativity& Critical Thinking
- Construction of Knowledge

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Multiple-Choice versus Selected-Response

Progressions for the Common Core State Standards in Mathematics (draft)

7 April 2011

Addition: Recording combined hundreds, tens, and ones on separate lines

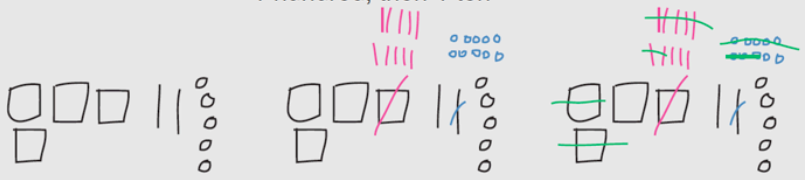
456	456	456	456
+167	+167	+167	+167
	500	500	500
		110	110
			13
			623

Addition proceeds from left to right, but could also have gone from right to left. There are two advantages of working left to right: Many students prefer it because they read from left to right, and working first with the largest units yields a closer approximation earlier.

Subtraction: Decomposing where needed first

decomposing left to right, 1 hundred, then 1 ten

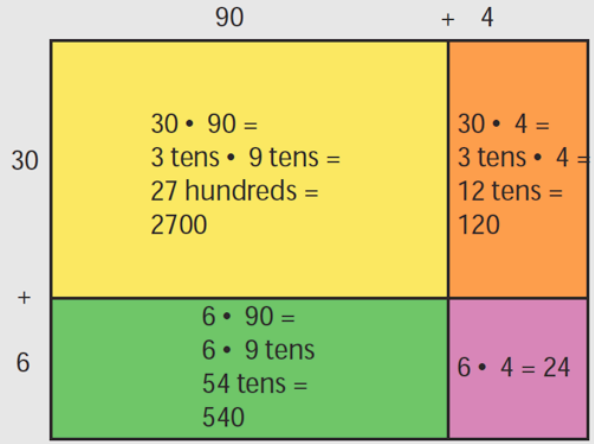
now subtract



$$\begin{array}{r} 425 \\ -\ 278 \\ \hline \end{array} \qquad \begin{array}{r} {}^{3}\not{4}{}^{1}\not{2}{}^{1}5 \\ \not{4}\not{2}\not{5} \\ -\ 278 \\ \hline \end{array} \qquad \begin{array}{r} {}^{3}\not{4}{}^{1}\not{2}{}^{1}5 \\ \not{4}\not{2}\not{5} \\ -\ 278 \\ \hline 147 \end{array}$$

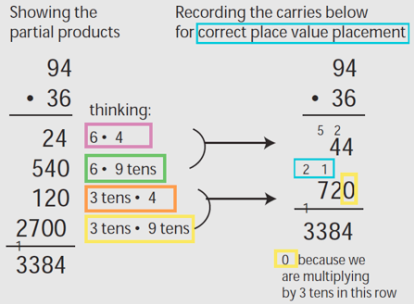
All necessary decomposing is done first, then the subtractions are carried out. This highlights the two major steps involved and can help to inhibit the common error of subtracting a smaller digit on the top from a larger digit. Decomposing and subtracting can start from the left (as shown) or the right.

Computation of 36×94 connected with an area model



The products of like base-ten units are shown as parts of a rectangular region.

Computation of 36×94: Ways to record general methods



These proceed from right to left, but could go left to right. On the right, digits that represent newly composed tens and hundreds are written below the line instead of above 94. The digits 2 and 1 are surrounded by a blue box. The 1 from $30 \times 4 = 120$ is placed correctly in the hundreds place and the digit 2 from $30 \times 90 = 2700$ is placed correctly in the thousands place. If these digits had been placed above 94, they would be in incorrect places. Note that the 0 (surrounded by a yellow box) in the ones place of the second line of the method on the right is there because the whole line of digits is produced by multiplying by 30 (not 3).

Multiple Algorithms Side-By-Side Connection to Prior Knowledge

Traditional

```
  1
 47
+35
 82
```

Partial Sums

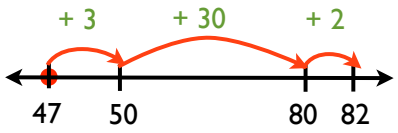
```
  47
 +35
  12   ← 5+7
+ 70   ← 30+40
  82
```

Decomposition

47 + 35
= 47 + 3 + 32
= 50 + 32
= 50 + 30 + 2
= 80 + 2
= 82

47 + 35
= 40+7+30+5
= 70 + 7+ 5
= 70 + 7+ 3+2
= 70 + 10 + 2
= 82

Open Number Line



2-Digit Multiplication

Traditional Algorithm

$$\begin{array}{r} {}^{1}\not{2}\,{}^{1}\not{3} \\ \times\ 14 \\ \hline 92 \\ +\ 230 \\ \hline 322 \end{array}$$

Partial Products

23

x 14

12

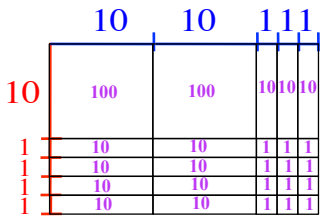
80

30

+ 200

322

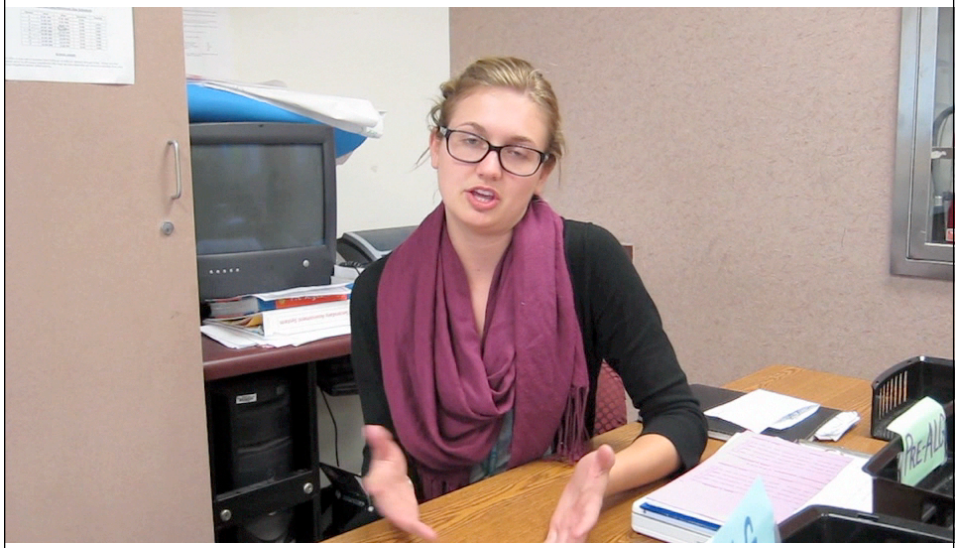
Area Models



Generic Rectangle

	20	3
10	200	30
4	80	12

Multiple Methods — Teacher Perspective



Overview of Smarter Balanced Mathematics Assessment Types

Selected Response Items (SR)

Constructed Response Items (CR)

Technology-Enhanced Items (TE)

Extended-Response Items (ER)

Multiple-Choice --- Selected-Response

Scoring Rubric

Responses to this item will receive 0-2 points, based upon the following:

2 points: YNYN The student has a solid understanding of 2/5 as well as the equivalent form of 2/5.

1 point: YNNN, YYNN, YYYN The student has only a basic understanding of 2/5. Either the student doesn't recognize an equivalent fraction for 2/5 or doesn't understand that all 5 parts must be equal-sized in figure 1b.

0 points: YYYY, YNNY, NNNN, NNYY, NYYN, NYNN, NYYY, NYNN, NNNN, NYNY, NNYN, NNNY The student demonstrates inconsistent understanding of 2/5 or answers "Y" to figure 1d, clearly showing a misunderstanding of what 2/5 means. Figure 1d is considered a "disqualifier" and an answer of "Y" to this part of the item would cancel out any other correct responses as "guesses" on the part of the student.

Selected-Response Items (SR)

- Can inform teachers more precisely in terms of what students know and are able to do.
- Mitigate the "guessing factor" of traditional selected-response items

Grade 5 Mathematics Sample SR Item C1 TF

MAT.05.SR.1.000NF.F.026 C1 TF

Sample Item ID:	MAT.05.SR.1.000NF.F.026
Grade:	05
Claim(s):	**Claim 1: Concepts and Procedures** Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 F: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
Content Domain:	Number and Operations – Fractions
Standard(s):	5.NF.5
Mathematical Practice(s):	2
DOK:	1
Item Type:	SR
Score Points:	1
Difficulty:	M
Key:	
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	Fractions may use any denominator that is a multiple of 2, 3, 5, and/or 7 and less than or equal to 100.
Notes:	Multi-part item

Your turn: 5.NF.5

Grade 5 Number and Operations–Fractions

For numbers 1a-1c, select Yes or No to indicate whether each fraction can be placed in the box to make a true inequality.

$$\frac{3}{4} \times \square > \frac{3}{4}$$

1a.	$\frac{12}{9}$	● Yes	○ No
1b.	$\frac{9}{9}$	○ Yes	● No
1c.	$\frac{9}{12}$	○ Yes	● No

Selected Response Items (SR) – Sample

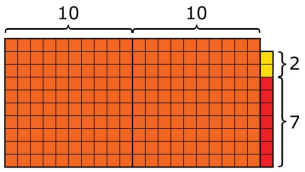
What problems can be represented by the incorrect responses?

A multiplication problem is shown below.

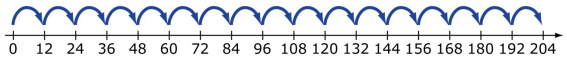
$$\begin{array}{r} 17 \\ \times 12 \\ \hline \end{array}$$

Which model(s) below could represent the solution to this problem?
Click all that apply.

Ⓐ

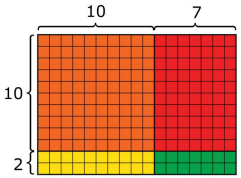


Ⓑ



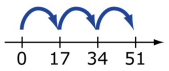
Ⓒ $(1 \times 1)+(1 \times 7)+(1 \times 2)+(2 \times 7)$

Ⓓ



Ⓔ $(17 \times 2)+(17 \times 1)$

Ⓕ



Mathematics Sample SR Item

SMARTER Balanced Assessment Consortium

Key and Distractor Analysis:

A Does not understand how to model multiplication of two two-digit numbers using area models.
B Correct
C Did not account for the values of the digits in the tens places.
D Correct
E Did not understand that the 1 represents 10 in the multiplication problem
F Showed multiplication of 17 and (1 + 2) instead of 17 and 12

Responses to this item will receive 0-2 points, based on the following:

2 points: B, D
1 point: Either B or D
0 points: Any other combination of selections.

Given:

$$(x + 4)^2 - (x - 2)(x + 4)$$

Select all the expressions below that are equivalent to the given expression.

A 24

B $2(x + 4)$

C $-2(x - 12)$

D $6(x + 4)$

E $(x + 4) - (x - 2)$

F $(x + 4)[(x + 4) - (x - 2)]$

$$(x+4)^2-(x-2)(x+4)$$

$$=\left[(x+4)-(x-2)\right]$$

$$=(x+4)[x+4-x+2]$$

$$=(x+4)[6]$$

$$=6(x+4)$$

Philip Gonsalves September 2012

Scoring Rubric for Multi-part Items:

The response to this item will receive 0–2 points, based on the following:

2 points: D, F

1 point: Only D or F

0 points: Any other combination of selections

A. squares each term separately in the first binomial, multiplies *x* terms separately and constants separately, then combines

B. squares each term separately in the first binomial and does not fully distribute negative into second polynomial

C. squares each term separately in the first binomial

D. Key

E. thinks $(x + 4)^2 - (x - 2)(x + 4)$ combines as $(x + 4)^2 - (x + 4) = (x + 4)$

F. Key

We have created and posted online sample SR items

Grade 5 Selected Response 3.NF.2

Which of these methods would be appropriate for finding $\frac{2}{3} + \frac{5}{12}$?

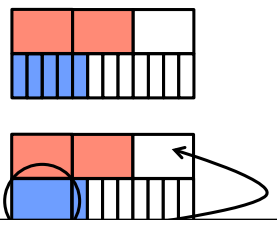
A

$$\frac{2}{3}+\frac{5}{12}$$
$$=\frac{2+5}{3+12}$$

B

$$\frac{2}{3} + \frac{5}{12}$$
$$=\frac{2}{3}\left(\frac{4}{4}\right)+ \frac{5}{12}$$
$$= \frac{8}{12} + \frac{5}{12}$$
$$= \frac{8+5}{12}$$

C



=

D

$$\frac{2}{3} + \frac{5}{12}$$
$$=\frac{2}{3}(4)+ \frac{5}{12}$$
$$= \frac{2}{12} + \frac{5}{12}$$
$$= \frac{2+5}{12}$$

E

$$\frac{2}{3} + \frac{5}{12}$$
$$=\frac{1}{3} + \frac{1}{3} + \frac{5}{12}$$
$$=\frac{4}{12}+\frac{4}{12} + \frac{5}{12}$$
$$= \frac{4+4+5}{12}$$

Scoring:
2 points: If selected B, C, and E
1 point: If selected any two of B, C, and E
0 points: All other combination of choices

Key and Distractor Analysis:

A. Students added the numerators and added the denominators. ("Added across")
B. Key. Correctly shows one way to create like denominators and add fractions.
C. Key. Correctly shows one way to model the sum on a bar model

Constructed Response Items (CR) – Sample

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Mathematics Sample CR Item

Sample Item ID:	MAT.04.ER.3.00NBT.E.037
Grade:	04
Primary Claim:	**Claim 3: Communicating Reasoning** Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Conceptual Understanding and Procedural Fluency Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Numbers and Operations in Base Ten
Secondary Content Domain(s):	
Assessment Target(s):	3 E: Distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in the argument, explain what it is. (DOK 2, 3, 4) 1 E: Use place-value understanding and properties of operations to perform multi-digit arithmetic. (DOK 1, 2)
Standard(s):	4.NBT.5
Mathematical Practice(s):	1, 2, 3, 4, 5
DOK:	3
Item Type:	ER
Score Points:	2
Difficulty:	H
Key:	See Rubric
Stimulus/Source:	
Target-specific Attributes (e.g., accessibility issues):	
Notes:	part of a PT item set

Constructed Response Items (CR) – Sample

Method W

23 × 49

20 × 9 = 180
3 × 9 = 27
20 × 4 = 80
3 × 4 = + 12
299

Method Z

23 × 49

Area Model

	40	+ 9
20	800	180
+ 3	120	27

Rectangle Sections

1
800
120
180
+ 27
1,127

Identify the method where Pablo made a mistake and explain what he should do to correct it.

Pablo made his mistake in Method W. The 4 represents 4 tens or 40 not 4. He needs to change the 80 to 800 and the 12 to 120.

Mathematics Sample CR Item

SMARTER Balanced Assessment Consortium

Sample Top-Score Response:

Pablo made a mistake when using Method W. He should have multiplied 20 and 3 by 40 instead of by 4. He made an place value error. Multiplying by 40 instead of 4 would have resulted in the same answer as when he used Method Z (1,127).

Scoring Rubric:

Responses to this item will receive 0-2 points, based on the following:

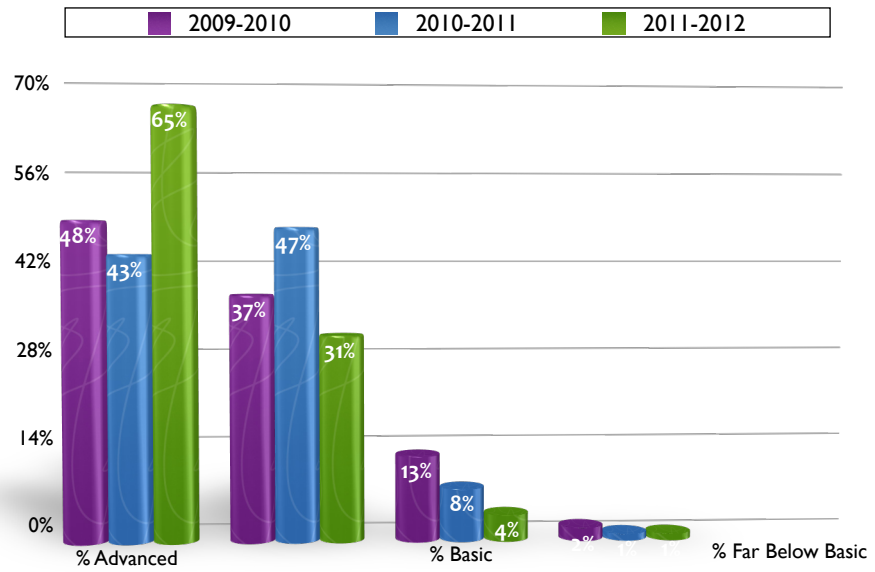
2 points: The student has a thorough understanding of how to multiply multi-digit whole numbers using more than one strategy to verify answers. The student indicates the place value error in Method W and explains how to correct the error.

1 point: The student has a partial understanding of how to multiply multi-digit whole numbers using more than one strategy to verify answers. The student indicates the place value error in Method W, but does not fully explain how to correct the error.

0 points: The student has little or no understanding of how to multiply multi-digit whole numbers using more than one strategy to verify answers. The student indicates that the error occurred in Method W, but does not identify the error correctly and includes an explanation that does not make sense mathematically. **OR** The student indicates an error in Method Z that does not exist.

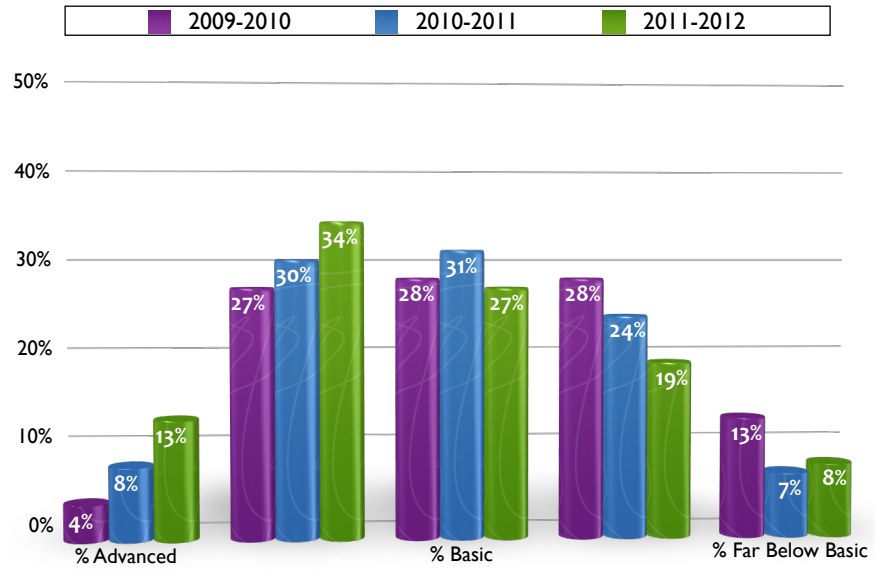
Multiple Methods — Student Perspective

AUSD Grade 7 Algebra I CST Data



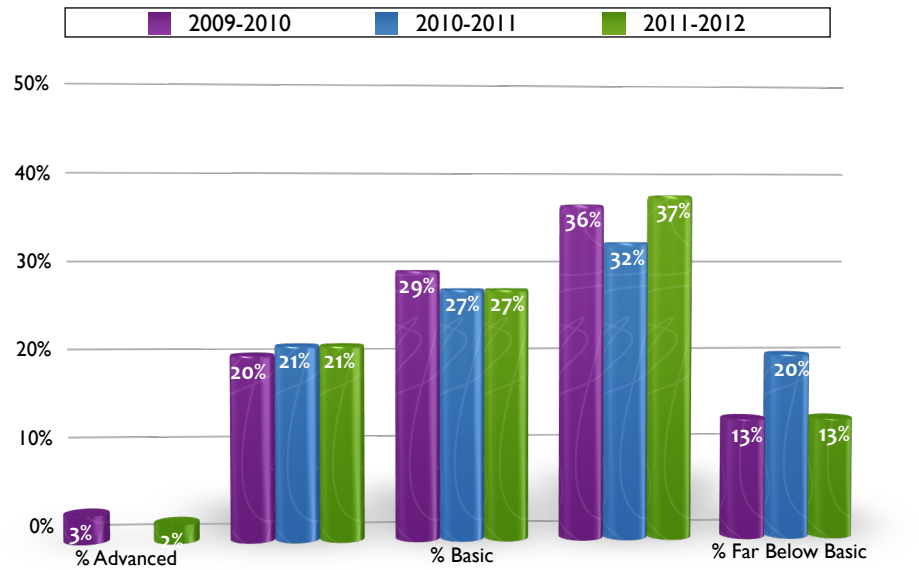
31

AUSD Grade 8 Algebra I CST Data



32

AUSD Grade 9 Algebra I CST Data



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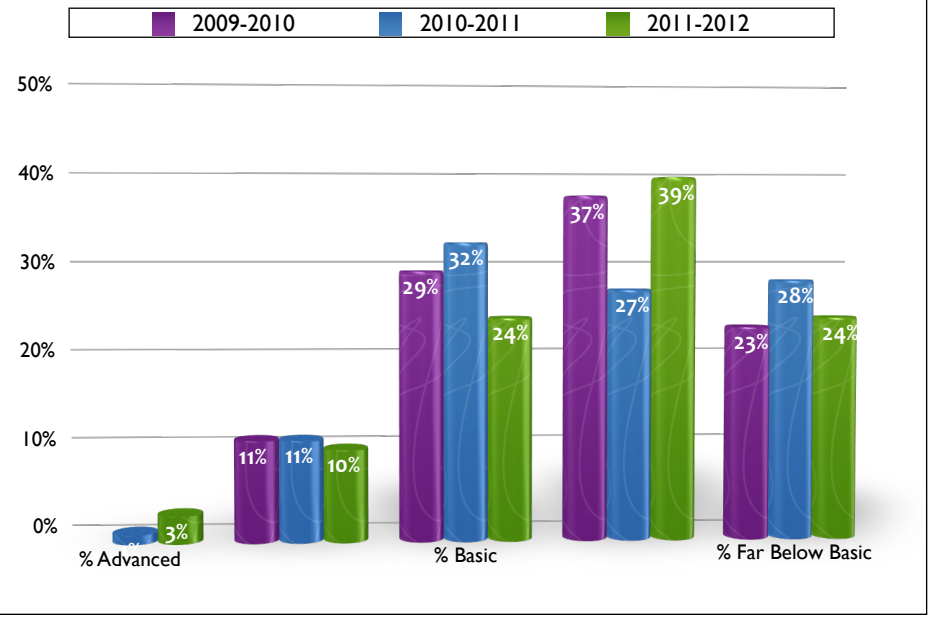
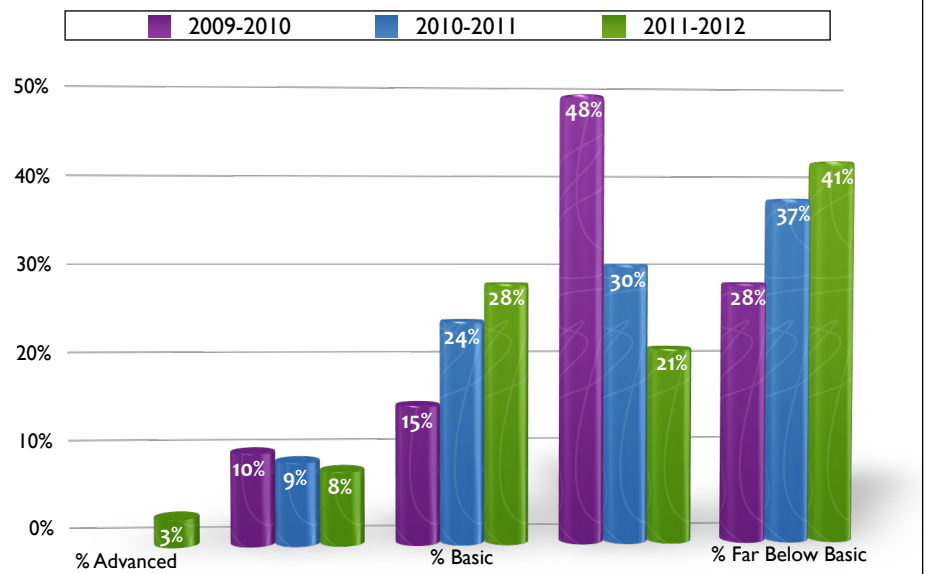
AUSD Grade 10 Algebra I CST Data



34

AUSD Grade 11 Algebra I CST Data



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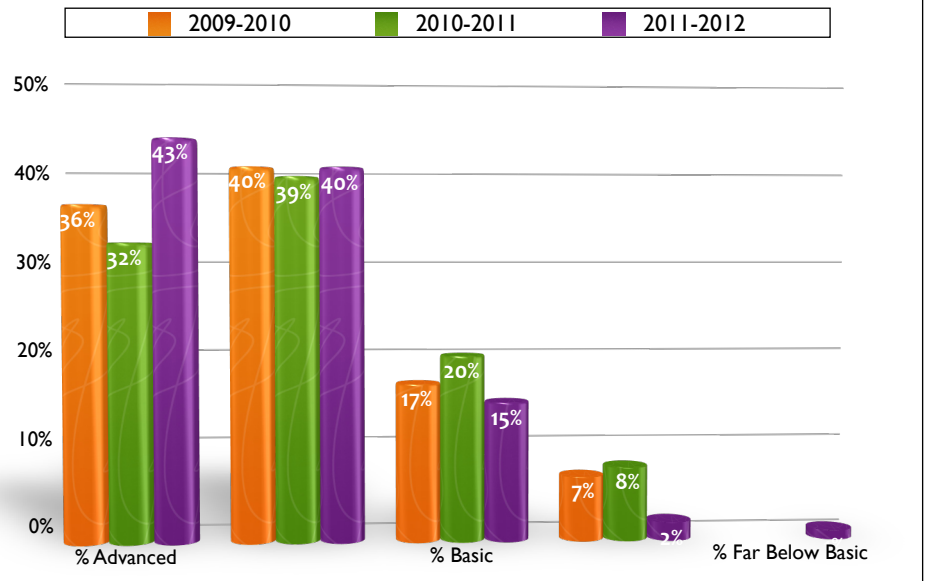
AUSD Grade 9 Algebra II CST Data



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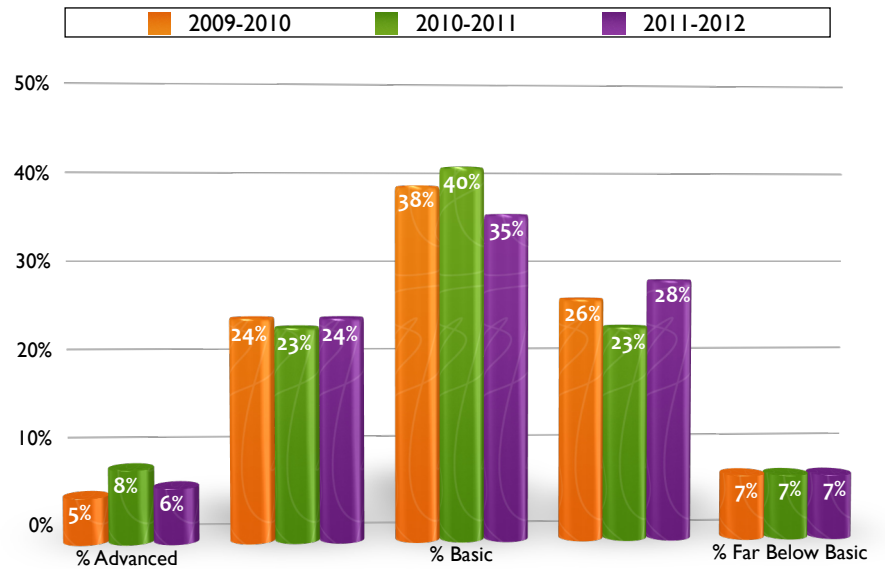
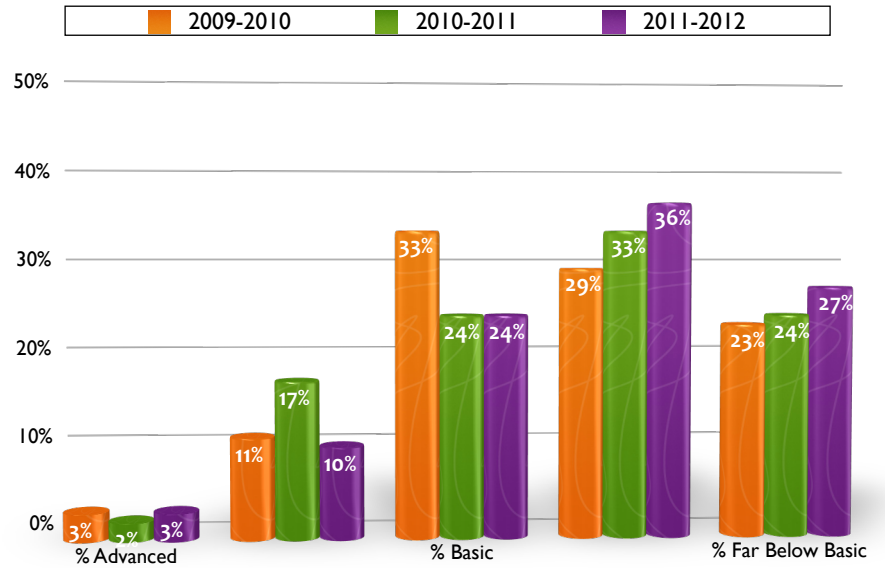
AUSD Grade 10 Algebra II CST Data



AUSD Grade 11 Algebra II CST Data



AUSD MAA Algebra II 2012 Data